

Generalized DGLAP evolution

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Abstract. We present progress in development of the truncated Mellin moments approach (TMMA). We show our recent results on the generalization of DGLAP evolution equations and discuss some their applications in spin physics.

1. Introduction

According to the factorization theorem, the cross sections for DIS reactions and some classes of hadron - hadron collisions can be expressed as convolution of two parts: a short-distance perturbative and a long-distance nonperturbative ones. The perturbative part, describing partonic cross sections at sufficiently high scale of the momentum transfer Q can be calculated within perturbative chromodynamics (pQCD). The non-perturbative part contains universal, process independent parton distribution functions $f(x)$ (PDF) and fragmentation functions $D_q^h(x)$ (FF), which can be measured experimentally. The evolution of these functions with the interaction scale Q^2 is again described with the use of the perturbative QCD methods. The standard DGLAP approach [1–3] enables one to calculate parton densities which characterize the internal nucleon structure at a given scale Q^2 when these densities are known for a certain input scale Q_0^2 . We have shown that also the truncated Mellin moments of the PDFs, $\int_z^1 x^{n-1} f(x) dx$, satisfy the DGLAP evolution equations and can be an additional tool in the QCD analysis of structure functions. The major advantage of the TMMA is a possibility to adapt theoretical QCD analysis to the experimentally available region of the Bjorken- x variable. In this way, one can avoid the problem of dealing with the unphysical region $x \rightarrow 0$ corresponding to the infinite energy of interaction. A number of important issues in particle physics, e.g., solving of the ‘nucleon spin puzzle’, quark - hadron duality or higher twist contributions to the structure functions refers directly to moments. Note that TMM, contrary to standard moments, may be directly extracted from the accurate (JLab) data by appropriate binning (keeping Q^2 fixed). These issues initiate a large number of experimental projects and theoretical studies as well. Below we present the generalization of DGLAP evolution equations within TMMA and discuss some applications in spin physics.

2. Truncated Mellin moments approach

The main finding of the TMMA is that the generalized truncated (cut) moments (CMM) obtained by multiple integrations as well as multiple differentiations of the original parton distribution also satisfy the DGLAP equations with the simply transformed evolution kernel [4–6]. A similar generalized evolution equation, with the correspondingly modified coefficient functions, can also

Table 1. CMM (first column) and the corresponding evolution kernels (second column).

| Generalized CMM | DGLAP kernel \mathcal{P} |
|---------------------------------------------------------------------------|------------------------------------|
| 1. $f(x)$ | $P(y)$ |
| 2. $x^n f(x)$ | $P(y) \cdot y^n$ |
| 3. $\int_z^1 dx x^{n-1} f(x)$ | $P(y) \cdot y^n$ |
| 4. $\int_z^1 z_k^{n_k-1} dz_k \dots \int_{z_2}^1 z_1^{n_1-1} f(z_1) dz_1$ | $P(y) \cdot y^{n_1+n_2+\dots+n_k}$ |
| 5. $f_\omega(z, n) = (\omega * f x^n)$ | $P(y) \cdot y^n$ |
| 6. $\left(-\frac{d}{dx}\right)^k [x^n f(x)]$ | $P(y) \cdot y^{n-k}$ |

be obtained for structure functions. In Table 1, we summarize the generalized CMM together with the correspondingly transformed DGLAP evolution kernels.

3. Applications of TMMA

Below we present examples of applications of TMMA to analysis of the Bjorken sum rule [7] and fragmentation functions.

3.1. Generalized Bjorken sum rule

For any normalized function $\omega(x)$, $\int_0^1 \omega(t) dt = 1$, one can construct generalized CMM $f_\omega(z, n)$ as a Mellin convolution with the function f , which obeys the DGLAP evolution equation with the rescaled kernel [8]:

$$f_\omega(z, n) = (\omega * f x^n) \equiv \int_z^1 \omega(z/x) f(x) x^n dx/x, \quad (1)$$

$$\mathcal{P}(y) = P(y) \cdot y^n \quad (2)$$

The special case of f_ω is suitable for the generalized Bjorken sum rule (BSR):

$$G_\omega(x, Q^2) = (\omega * g_1^{NS})(x). \quad (3)$$

G_ω has the same evolution kernel as g_1^{NS} and the generalized BSR is equal to the ordinary BSR:

$$\int_0^1 G_\omega(x, Q^2) dx = \int_0^1 g_1^{NS}(x, Q^2) dx = \text{BSR}. \quad (4)$$

The corresponding cut first moments of G_ω go to the BSR limit as the cut point x_0 goes to zero. This allows one to study behaviour of the generalized cut moments near $x_0 = 0$ and estimate the value of the BSR from the cut integrals $\int_{x_0}^1 G_\omega(x, Q^2) dx$ at $x_0 \neq 0$. The attempts for the case $\omega(t) = n t^{n-1}$ are shown in Fig. 1 and can be tested experimentally. We have also calculated contributions to the BSR itself and compared them to the experimental data. In Fig. 2 we compare TMMA predictions for the contributions to the BSR to recent COMPASS data. One

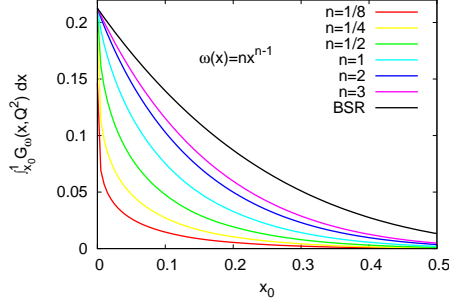


Figure 1. The cut first moments of the generalized CMM $G_n(3)$, where $\omega = n t^{n-1}$, for different n versus the cut point x_0 .

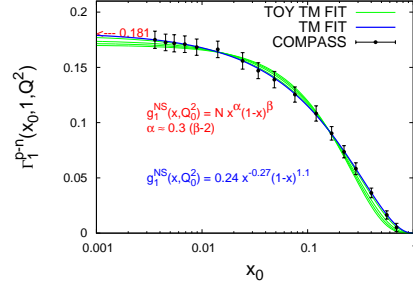


Figure 2. Contributions to the Bjorken sum rule obtained within TMMA. Comparison to COMPASS data.

Table 2. Truncated contributions to the Bjorken sum rule with use different input parameterizations. Comparison with experimental data.

| N | INPUT | x -range | Q^2 [GeV ²] | Γ_1^{NS} | EXP Γ_1^{NS} |
|----|-------------------------------|------------|---------------------------|-----------------|--------------------------------|
| 1. | $(1-x)^3$ | | | 0.161 | HERMES |
| 2. | $x^{-0.2}(1-x)^3$ | 0.021-0.9 | 5 | 0.149 | $0.1479 \pm 0.0055 \pm 0.0142$ |
| 3. | $x^{-0.4}(1-x)^3$ | | | 0.131 | |
| 4. | $(1-x)^3$ | | | 0.177 | COMPASS |
| 5. | $x^{-0.2}(1-x)^3$ | 0.004-0.7 | 3 | 0.173 | $0.175 \pm 0.009 \pm 0.015$ |
| 6. | $x^{-0.4}(1-x)^3$ | | | 0.163 | |
| 7. | $x^\alpha(1-x)^\beta$ | | | 0.168- | COMPASS |
| 8. | $\alpha \approx 0.3(\beta-2)$ | 0.0025-0.7 | 3 | 0.170 | 0.170 ± 0.008 |

can see that simple input parametrization 7, where $\alpha = 0.3(\beta - 2)$ can satisfactorily reproduce the experimental data. This relation between α and β , together with the positivity constraint can provide knowledge on the small- x behaviour of the polarized structure function g_1^{NS} . From our analysis the favoured small- x behaviour of g_1^{NS} is x^α , where $\alpha = -0.2 \div -0.3$. Table 2 contains the truncated contributions to the Bjorken sum rule in the experimentally available x -region,

$$\Gamma_1^{p-n}(x_1, x_2, Q^2) = \int_{x_1}^{x_2} g_1^{NS}(x, Q^2) dx \quad (5)$$

obtained for different input parametrizations

$$g_1^{NS}(x, Q_0^2) = N x^{a_1} (1-x)^{a_2}. \quad (6)$$

Our predictions are compared with the HERMES [9] and COMPASS [10] data.

3.2. Fragmentation functions

Finally, it is worthy to mention that TMMA can be very useful in analysis of the hadron fragmentation functions as in the small- x region behaviour of FF is known very poorly. In this way, one can restrict the analysis to well determined models for $x \geq x_0$. FF also obey the corresponding DGLAP evolution and their CMM can provide new insight into the hadron

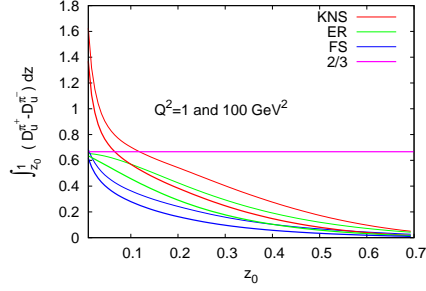


Figure 3. TMM of the pion FF, $Q^2 = 1, 100 \text{ GeV}^2$ (thin, thick). Inputs: KNS [11], ER [12], FS [13]

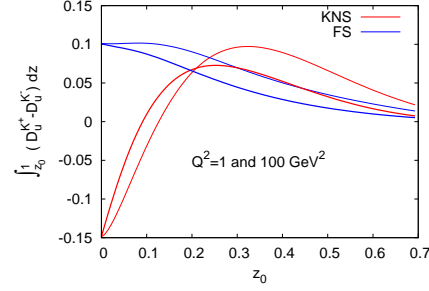


Figure 4. TMM of the kaon FF, $Q^2 = 1, 100 \text{ GeV}^2$ (thin, thick). Inputs: KNS [11], FS [13]

structure.

In Figs. 3,4 we present evolution of the truncated moments of FF contributing to the quark charge conservation,

$$\sum_h Q_h \int_0^1 dz D_q^h(z, Q^2) = Q_q, \quad (7)$$

where D_q^h denotes a fragmentation function of the hadron h from a parton q and Q_h, Q_q are the charges of the hadron h and parton q . We use different parametrization of the FF at the initial scale [11–13]. The excess of the obtained moments for pions, providing the main contributions to sum rule, over the charge conservation values at $z_0 \sim 0.2$ may be considered as a support of inapplicability of independent fragmentation picture at this region. This fact is also exhibited in the large differences in the presented predictions in this region, depending on the input parametrization, for both pions and kaons. Therefore TMM approach can be a natural tool also in the study of the fragmentation functions, requiring further investigations.

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